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EXPERIMENTAL STUDY OF THE BEHAVIOR OF QUADRUPLE TIME CORRELATION FUNCTIONS FOR THE LONGITUDINAL VELOCITY FLUCTUATION IN DEVELOPED TURBULENT PIPE FLOW

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1. Formulation of the Problem. One method of closing the system of equations describing turbulent flow is that of using the relation between the fourth- and second-order correlation moments according to the hypothesis of zero values of fourth-order cumulants [1, 2]:

$$\langle u_i u_j u'_m u'_n \rangle - \langle u_i u_j \rangle \langle u'_m u'_n \rangle = M_{ij}^{mn} - M_{ij} M^{mn} \cong M_i^m M_j^n + M_i^n M_j^m \qquad (1.1)$$

(the subscripts refer to the first time point and the superscripts to the second). The same hypothesis can be expressed in terms of the Fourier transforms corresponding to the fourth- and second-order moments [2, 3]:

$$E_{ij}^{mn}(\mathrm{k}) = \int_{-\infty}^{\infty} E_{im}(\mathrm{k}-\mathrm{k}')\, E_{jn}(\mathrm{k}')\, d\mathrm{k}' + \int_{-\infty}^{\infty} E_{in}(\mathrm{k}-\mathrm{k}')\, E_{jm}(\mathrm{k}-\mathrm{k}')\, d\mathrm{k}'.$$

The true probability-density distribution of velocity fluctuations is substituted by a model probability density function that could be yet a good approximation for calculating certain moments if the region of its negative values has little effect on the results of calculating these moments.

Millionshchikov's hypothesis was experimentally verified in [4] for isotropic flow behind a grid. The experimental data were found to satisfy this hypothesis within the measurement error.

The present paper is aimed at comparing experimental relations for developed turbulent pipe flows.

2. Experimental Setup and Measuring Equipment. The setup channel is a straight-line round pipe with diameter $d = 0.06$ m. The first part of the channel 6 m long is immovable and serves to form a developed symmetric turbulent flow that corresponds to a specified Reynolds number. The second part of the channel 1.5 m long can be rotated around the longitudinal axis. The flow is created by the air supply from a high-pressure pipeline through a heater and a receiver with a converging nozzle (contoured according to Vitoshinskii's relation) which ensures a flow-contraction ratio of $1:12$. The room temperature is maintained in flow within 0.1°. The flow temperature is controlled by the thermocouple connected to a VK 2-20 voltmeter.

Hot-wire anemometer equipment of DISA Company (series M) was used to measure the flow velocity. Signal linearization was performed. A 55P11 straight general-purpose probe (wire length 1.25 mm and diameter 5 μm) was placed at various distances from the axis along the horizontal radius $\bar{r}$ of the cross section of the pipe. The wire was directed vertically along the circumferential component of the flow velocity. The probe is not sensitive to the velocity component along the wire. Therefore, only the signal from the velocity component normal to the wire is taken into account. The wire was placed at a distance of 15 mm from the exit section inside the pipe $\bar{x}_1$.

The time realizations of the signals were registered by a digital tape-recorder, and their processing was performed on a Plurimat-S computer. To obtain one value of the time correlation functions, 153,000 signal samples with a sampling frequency of 10 kHz were used.

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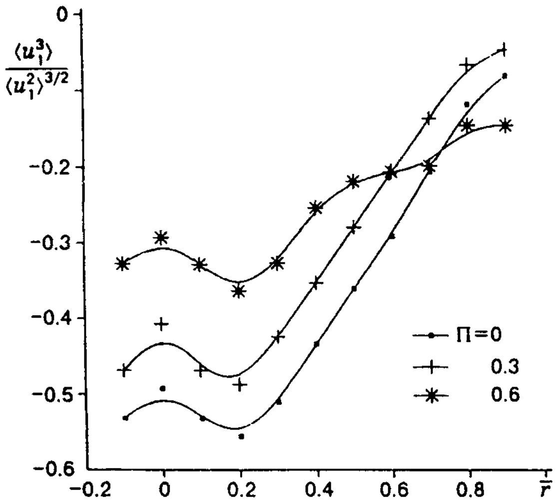

Fig. 1

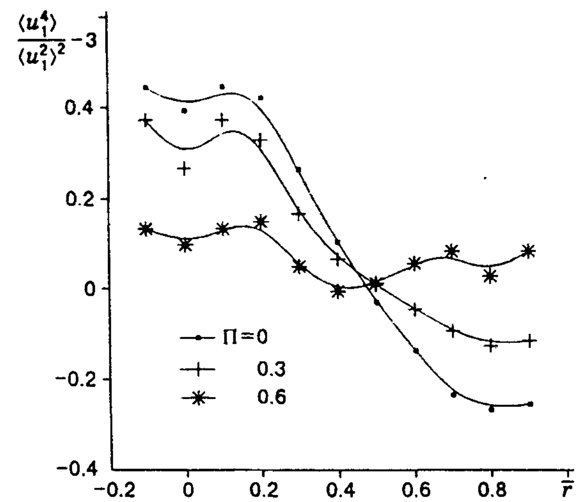

Fig. 2

The data for the mean velocity $U_0 = 10$ m/sec on the axis are presented below. The Reynolds number is $\mathrm{Re} = U_0 d/\nu = 4.2 \cdot 10^4$ and the kinematic viscosity is $\nu = 1.42 \cdot 10^{-5}$ m^2/sec. The flow swirl level is characterized by the parameter $\Pi = V_\varphi^0/U_0$, where V_φ^0 is the circumferential rotation velocity of the pipe wall.

One-point moments of the longitudinal-velocity fluctuation for moments of various orders were measured for verification of the technique. The results are compared with the literature data: the data of [5–10] for second-order moments, [5, 7, 9] for third-order moments, [5, 7] for fourth-order moments, and [7] for fifth- and sixth-order moments. The comparison and the experimental results in tabulated graphical forms are presented in [11–13].

An x-array probe was used to measure the circumferential component of the mean velocity and the shear stress.

With increasing flow swirl, the axial-velocity profiles become more and more full, which is due to reduction in turbulent friction. The circumferential velocity is negligible in the central part of the flow and increases abruptly near the pipe walls. Increasing swirl reduces the values of all moments, skewness and kurtosis coefficients practically in the entire cross section of the flow within the dimensionless radius of from zero to 0.8. The skewness is shown in Fig. 1 as a function of the pipe radius for various values of the swirl parameter. Figure 2 shows curves of the kurtosis coefficient. The fifth- and sixth-order moments exhibit a similar behavior. The probability distributions approach the Gaussian curve. The agreement of data is fairly satisfactory and is indirect evidence that the accuracy of the measurements is acceptable.

3. Time Correlation Functions. The quadruple [of the type M_{11}^{11} ($\bar{r}$, $\bar{x}_1$, τ)] and double time correlation functions were examined. Direct use of Millionshchikov's hypothesis in the relation between correlation functions is known to lead to an error for $\tau = 0$, because the true value of the kurtosis coefficient in an inhomogeneous, non-isotropic flow differs from the Gaussian one. However, the hypothesis can be used in spectral form only for the Fourier transform of the cumulant function, i.e., for expression (1.1). Thus, an attempt is made to use Millionshchikov's hypothesis in somewhat changed form for the moment functions as well. The expression $M_{11}^{11}(\tau) - (M_{11}(0))^2$ is normalized to the value for $\tau = 0$, and the approximate expression for the right-hand side $2(M_1^1(\tau))^2$ is normalized to the approximate term $(T_0 - 1)(M_{11}(0))^2 = 2(M_{11}(0))^2$. Here $T = \langle u_1^4\rangle/\langle u_1^2\rangle^2$, and for the Gaussian distribution $T_0 = 3$. Then one can find $[M_{11}^{11}(\tau)]_0$ from the second-order moments $M_1^1(\tau)$ and the kurtosis coefficient measured at the point under consideration:

$$[M_{11}^{11}(\tau)]_0 - (M_{11}(0))^2 \cong (T - 1)(M_1^1(\tau))^2. \tag{3.1}$$

The error for intermediate values of τ, or, what is the same, for intermediate frequency values that

TABLE 1

$\bar{r}$	$\langle U\rangle$, m/sec	$\langle u_1^2\rangle$, m^2/sec^2	T	$10^4\eta$, m	$10^3\lambda$, m	$10^2\Lambda$, m	f_η, Hz	f_Λ, Hz
$\Pi = 0$								
0	10	0.128	3.39	1.71	2.64	0.97	9300	163
0.2	9.78	0.178	3.42	1.63	2.81	1.22	9500	127
0.4	9.33	0.286	3.10	1.46	2.88	1.53	10200	97
0.6	8.69	0.425	2.86	1.28	2.65	1.53	10800	91
0.8	7.76	0.579	2.73	1.08	2.25	1.28	11400	96
0.9	6.98	0.670	2.75	0.94	1.81	0.93	11800	119

$\bar{r}$	$\langle U\rangle$, m/sec	$\langle u_1^2\rangle$, m^2/sec^2	T	$10^4\eta$, m	$10^3\lambda$, m	$10^2\Lambda$, m	$\langle V_\varphi\rangle$, m/sec
$\Pi = 0.3$							
0	10	0.059	3.27	2.03	2.72	0.85	0
0.2	9.88	0.090	3.33	1.89	2.85	1.05	
0.4	9.40	0.166	3.07	1.59	2.74	1.20	0.15
0.6	8.64	0.276	2.95	1.30	2.33	1.08	0.40
0.8	7.32	0.407	2.87	1.04	1.82	0.82	1.0
0.9	6.28	0.496	2.88				1.5

$\bar{r}$	$\langle U\rangle$, m/sec	$\langle u_1^2\rangle$, m^2/sec^2	T	$10^4\eta$, m	$10^3\lambda$, m	$10^2\Lambda$, m	$\langle V_\varphi\rangle$, m/sec
$\Pi = 0.6$							
0	10.3	0.035	3.10	2.34	2.81	0.77	0
0.2	10.2	0.050	3.15	2.20	2.96	0.93	
0.4	9.90	0.086	2.99	1.85	2.82	1.03	0.14
0.6	9.11	0.152	3.06	1.42	2.20	0.82	0.51
0.8	7.62	0.286	3.03	1.01	1.46	0.50	1.51
0.9	6.38	0.390	3.08	0.80	1.02	0.30	2.7

correspond to the energetic region of the spectrum, can be determined using the relations for spectral distributions. Figure 3 shows the normalized spectral frequency distributions for $\bar{r} = 0$, 0.2, 0.4, 0.6, 0.8, and 0.9 (curves 1–6) and $\Pi = 0$ and 0.3 for fourth-order moments $M_{11}^{11}(\tau) - (M_{11}(0))^2$:

$$\Phi = \frac{E_{11}^{11}(f)}{(T-1)\int\limits_{-\infty}^{\infty} E_{11}(f-f')E_{11}(f')\,df'}.$$

The data on the flow are listed in Table 1. Taylor's hypothesis is used: the wave number is $k = 2\pi f/\langle U\rangle$,

$$\hat{k} = k\eta = \frac{2\pi f}{\langle U\rangle}\eta = f/f_\eta, \quad f_\eta = \frac{\langle U\rangle}{2\pi\eta}, \quad f_\Lambda = \frac{\langle U\rangle}{2\pi\Lambda},$$

where Λ is the longitudinal integral correlation scale, λ is Taylor's microscale, and η is Kolmogorov's microscale. One can see that, within an error of $\pm 10\%$ in the energetic frequency range, the experimental spectral distributions in thus normalized form agree fairly well with those calculated using formula (3.1) with Millionshchikov's approximation. As is seen from the data for one-point functions in Fig. 1, the kurtosis coefficient $(T-3)$ decreases by a factor of 3–4 in the flow at the exit of the rotating section of the pipe, and the error in determining the fourth-order moment also decreases.

Figure 4 shows the measured functions F_1 and F_2 (F_2 is an approximate dependence shown by the upper curve in each set) versus time for radial points $\bar{r} = 0$, 0.2, 0.4, and 0.6 (at $\Pi = 0$ and 0.3):

$$F_1 = \frac{(M_1^1(\tau))^2}{(M_{11}(0))^2}, \quad F_2 = \frac{M_{11}^{11}(\tau) - (M_{11}(0))^2}{M_{11}^{11}(0) - (M_{11}(0))^2}.$$

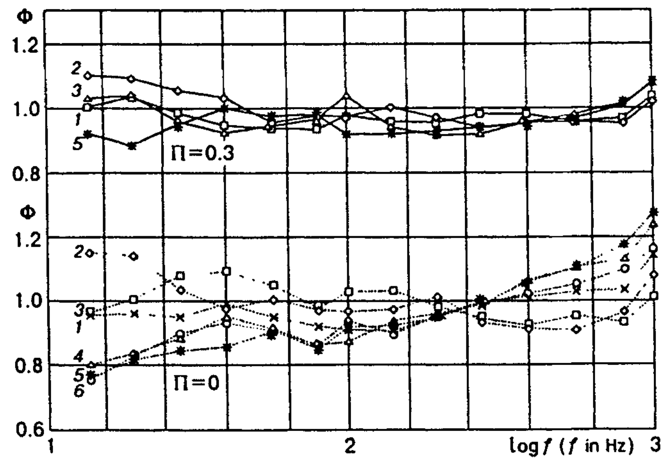

Fig. 3

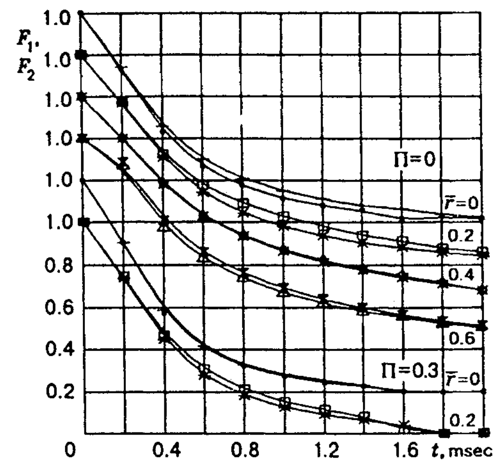

Fig. 4

As the value of $(T - 3)$ decreases, the difference between the curves becomes smaller. The large relative difference is retained for large times for which the function values are small. The difference between the moments $M_{11}^{11}(\tau)$ and $[M_{11}^{11}(\tau)]_0$ is within 20% for $\Pi = 0$ ($\bar{r} = 0$ and 0.8) and decreases to 5% for flow regimes with rotation of the pipe section.

Thus, it is found that

— when the pipe is rotated, the skewness and kurtosis coefficients characterizing the deviation of the distribution function from the Gaussian curve decrease in absolute value;

— in the developed turbulent pipe flow the fourth-order moments differ from Millionshchikov's approximations to the same extent as the kurtosis coefficient differs from the Gaussian value;

— an attempt to represent the spectral dependences for the time correlation functions in a special example with normalization using the true values of kurtosis in the flow leads to an approximate representation of the fourth-order time correlation moment using the second-order time correlation moment and the true value of the kurtosis coefficient as an expression with correct limit values at zero and large values of time and with an error of about 10–20% in the intermediate interval.

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